

Strategies and Rubrics for Teaching Chaos and Complex Systems Theories as Elaborating, Self-Organizing, and Fractionating Evolutionary Systems

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ABSTRACT

To say Earth systems are complex, is not the same as saying they are a complex system. A complex system, in the technical sense, is a group of “agents” (individual interacting units, like birds in a flock, sand grains in a ripple, or individual units of friction along a fault zone), existing far from equilibrium, interacting through positive and negative feedbacks, forming interdependent, dynamic, evolutionary networks, that possess universality properties common to all complex systems (bifurcations, sensitive dependence, fractal organization, and avalanche behaviour that follows power-law distributions.)

Chaos/complex systems theory behaviors are explicit, with their own assumptions, approaches, cognitive tools, and models that must be taught as deliberately and systematically as the equilibrium principles normally taught to students. We present a learning progression of concept building from chaos theory, through a variety of complex systems, and ending with how such systems result in increases in complexity, diversity, order, and/or interconnectedness with time—that is, evolve. Quantitative and qualitative course-end assessment data indicate that students who have gone through the rubrics are receptive to the ideas, and willing to continue to learn about, apply, and be influenced by them. The reliability/validity is strongly supported by open, written student comments.

INTRODUCTION

Two interrelated subjects are poised for rapid development in the Earth sciences. One is the application of chaos and complex systems theories, and the other is an expanded repertoire of theories about how Earth systems evolve—including theories compatible with but different from Darwinian evolutionary theory. These have the prospect of fundamentally changing the way we think about the evolutionary history of natural systems, but only if we develop systematic strategies for introducing and teaching these concepts.

Fichter, Pyle, and Whitmeyer (2010) posit that Earth systems, like many other natural systems, evolve by three evolutionary mechanisms: elaboration, fractionation, and self-organization, operating either individually or in concert within the same system. Each of these is best understood and explained as a complex system, in the technical sense of that phrase (explicated below). Elaborating evolution—subsuming biological evolution as a special case—begins with a seed, an ancestor, or a randomly generated population of agents, and evolves by generating, and randomly mutating, a large diversity of descendants which are evaluated by an external fitness function. The inclusive mechanism is the General Evolutionary Algorithm: (1) elaborate diversity, (2) selecting from among that diversity, and (3) amplify the result, (4) repeat. Self-organizing evolution begins with an initial state of random agents that through the application of simple rules (e.g. an algorithm, or chemical/physical laws) evolves a system of ordered structures, patterns, and/or connections without control or guidance by an external agent or process; that is, pulls itself up by its own boot straps. Fractionating evolution begins with a complex parent which is physically, chemically, or biologically

divided into fractions through the addition of sufficient energy because of differences in the size, weight, valence, reactivity, etc. of the component particles.

There are impediments to incorporating these ideas in the discipline and in the classroom. One impediment is the dominance of linear/equilibrium thinking and training in our schools (Fichter, Pyle, Whitmeyer, 2008). Teaching chaos/complex systems principles requires students be familiar with mathematical principles, techniques, and properties not yet systematically taught. A second impediment is the inconsistent and ambiguous use of the terms “complex” and “system.” A third impediment is the domination of biological evolutionary theory as the only systematic mechanism for evolutionary change. Finally, a fourth impediment is the absence of rubrics for introducing chaos/complex systems theories and modelling techniques in class rooms. This paper addresses all these issues and develops a systematic, theoretically coherent, and practical set of definitions, concepts, models, and rubrics for teaching ideas of complex evolutionary systems at the introductory level. We also include assessment results of the level to which students’ dispositions or habits of mind have been influenced by these methods for teaching chaos/complex evolutionary systems.

TEACHING EVOLUTION THROUGH CHAOS/COMPLEX SYSTEMS THEORY

All evolutionary systems are complex systems in the technical sense of that term (described below). It is possible to introduce elaborating, self-organizing, and fractionating evolution without the use of complex systems theory. Science has been doing it for a long time. Biological evolution is a complex system, but, until recently has not been thought of or modelled as a complex system. Likewise, fractionating evolution of petrographic systems using AFM or phase diagrams, or sediment evolution using QFL or Q/FL/Matrix diagrams has been

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a mainstay of the geosciences. We have been talking about evolution all along, but because we have modelled these as equilibrium systems there has been a conceptual block to thinking of them in any terms other than descent to equilibrium—which according to the definitions of evolution is not really evolution.

At the equilibrium level of thinking, elaborating, self-organizing, and fractionating evolution are not much more than new terms applied to old well-known ideas. They may allow us to think about Earth processes in a different context, but not much else. In science, though, theory should provide explainability, and equilibrium thinking cannot provide the explanation for the properties we observe in evolutionary systems (called *universality* properties in chaos /complex systems theory), such as, but not restricted to, bifurcations, sensitive dependence, avalanches of changes following power law distributions, with fractal organization and dynamic behaviour as strange attractors that often exhibit bi-stable (hysteresis) behaviour. What is common to, and binds together all evolutionary processes are these non-equilibrium universality properties. If students do not know what these phenomena are, and how they are related to each other, and what they tell us about the pulse of dynamic systems, then the idea of a complex system will be opaque. It is the recognition that all evolutionary systems are non-equilibrium systems and evolve to complexity with a whole new, integrated set of properties and behaviors that has the potential to transform our teaching of evolutionary Earth systems to a new paradigm.

What ARE Chaos/Complex Systems?

"When I use a word it means exactly what I want it to mean, neither more nor less" (Humpty Dumpty in *Through the Looking Glass*.) The terms "complex" and "system" have a variety of vernacular and technical meanings, which makes use of the phrase "complex system" ambiguous, and open to (mis)interpretation. *System*, for example, may be used in everyday speech as, "It's the system." "You can't beat the system," "You hav'ta play the system." We also have school systems, the Federal Reserve System, the Global Positioning System, operating systems, and the solar system. Some of these may behave as complex systems too, but it is not overtly obvious which ones, and some are specifically engineered to not behave as a chaos/complex system.

To say Earth systems are complex is not the same as saying they are a complex system. Chaos/complex systems theory is not about linear systems that may behave complexly on their way to equilibrium (Fichter, Pyle, Whitmeyer, 2008). Further confusion emerges when we ask questions such as, "Is a system complex just because it has a bunch of parts, or is behaviour important?" (A car has a bunch of parts, but its behaviour is simple, and it is engineered to not be a complex system in the technical sense of that phrase). "Or, simple parts that have complex behaviour?" (For example, the logistic system described below which is very simple but with exceedingly complex behaviour.) In chaos/complex systems theory a system of very few parts can exhibit very complex behaviour, while very elaborate systems may

appear simple, and still be complex systems. The answers to these questions are not always straightforward, and intuition may not always be an accurate guide. Considering the complexity of uses of both terms "complex" and "system" we must be as precise as we can, especially when we conjoin them as "complex system."

When we say evolutionary systems are complex we are using the terms in specific ways not to be confused with their other uses or meanings. Herbert (2006) provides a clear description of the variety of properties and behaviors of complex systems as we use the term here. Mitchell and Newman (2002) also concisely describe the properties of complex systems, using biological systems as an example, with an online version. More expansive descriptions of chaos/complex systems as applied to many phenomena are described in Waldrop (1992), Johnson (2002), Sole' (2002), and Strogatz (2004), among many others. For mathematical treatments of chaos/complex systems and their applications see Williams (1997), Strogatz (2001), Bar-Yam (2003), Turcotte (1997); there are many others.

There is as yet no universally agreed upon definition of a complex system, although there is large consensus about the properties and behaviors they possess. We begin with this generalized description of a complex system: *A complex system is a group of "agents" (individual interacting units, like birds in a flock, sand grains in a ripple, or the individual units of friction along a fault zone), existing far from equilibrium, interacting through positive and negative feedbacks, forming interdependent, dynamic, evolutionary networks, that are sensitive dependent, fractally organized, and exhibit avalanche behaviour (abrupt changes) that follow power-law distributions.* Complex systems produce large-scale behaviors that are not easily predicted from knowledge of the behaviors of the individual components themselves; i.e. they are emergent properties. Interactions among the components take on a variety of forms, many of which are evolutionary. In a complex system the interactions—the flows of energy and materials—are more important than the components themselves. These systems also exhibit the universality properties and behaviors listed above, and elaborated on below.

Why Chaos/Complex Systems May Be Difficult to Understand

Herbert (2006) and Raia (2005, 2008) have analysed some of the reasons why students have difficulty understanding complex systems. We agree that these difficulties are real. Yet, through trial and error we believe we have developed rubrics that help students understand chaos/complex systems. It began about 15 years ago when JMU faculty member Steve Baedke and the senior author began teaching a 1 credit course in Artificial Life as a venue to complex systems thinking. Since that was the sole goal of the course, we encountered class by class the difficulties and misconceptions students had, while continuously adjusting how we developed the ideas. One big advantage of the course was that it was mostly based on computer lab experiments so instead of just lecturing about the behaviors of complex systems students experimented and played with them, and the instructors

saw their confusions firsthand.

Normally, when we teach geoscience concepts we rely on prior-taught science and math concepts—often learned at the middle and high school levels—without feeling the necessity of re-teaching them in our courses (although as we all know, we sometimes have to anyway). However, these prior-taught concepts are virtually always going to be linear, equilibrium principles.

Assaraf and Orion (2005), in reviewing the literature, suggest that there are eight components that underscore students' understanding of Earth systems thinking:

1. The ability to identify the components of a system and processes within the system;
2. The ability to identify relationships among the system's components;
3. The ability to organize the systems' components and processes within a framework of relationships;
4. The ability to make generalizations;
5. The ability to identify dynamic relationships within the system;
6. Understanding the hidden dimensions of the system;
7. The ability to understand the cyclic nature of systems; and
8. Thinking temporally: retrospection and prediction.

Through their research with middle school students, they determined that students are often lacking in their understandings of each of these components. In tracking students' learning towards these components, they found a strong hierarchical component, such that 70% of students could identify system components and processes, while only about half could identify dynamic relationships between system components. Less than a third of students could identify networks of relationships, make generalizations, or suggest predictions of the system. This suggests that there is an uphill struggle in students' understanding of Earth systems, and without a consideration of a dynamic cyclicity in Earth systems, or the understanding of a system as a network of relationships, the highest levels of student understanding may not be reached.

Generally, equilibrium thinking does not naturally and easily incorporate or develop some of the eight points Assaraf and Orion (2005) outline above, especially points 2, 3, 5 and 6. On the other hand, the kind of thinking required for the list above is natural to chaos/complex systems theory. Yet, few or none of our students enter our classes having even heard of, let alone knowing, something about chaos/complexity concepts. Yet, chaos/complex systems theory behaviors are explicit, with specific properties that have their own set of assumptions, approaches, cognitive tools, and models that are antithetical to linear/equilibrium systems and, in fact, lead in the opposite direction from them. The principles of chaos/complex systems must be taught as deliberately and systematically as the equilibrium principles normally taught to students; as, say, the systematic training from

pre-algebra and geometry to algebra, and algebra to pre-calculus to calculus, a progression that has had decades of development, supported by extensive experience and research, infused into the academy. No wonder then, as Herbert states (2006, p 100), "It is likely that students have difficulties in understanding earth systems of even modest complexity, predicting future system behavior in a variety of scenarios, and reasoning correctly about complex environmental issues because of misconceptions, inaccuracies, or incompleteness in their mental models of these systems."

The difficulties students face with understanding chaos/complex systems is the absence of a learning progression of deliberate concept building from first principles, unfamiliarity with—or vernacular understanding of—much of the terminology, and no rubrics to systematically build the concepts and terminology in a logical way that gives it staying power. Acknowledging that it will be a long time before students enter college with anywhere near the systematic preparation to understand chaos/complex systems they now have for basic math and science, the only remaining choice is for us to present chaos/complex systems ideas in our own classes. Actually, the math preparation students have is adequate; it is mostly algebra. A greater problem is the science training that emphasizes linear systems that evolve to equilibrium. It is likely that part of the difficulty our students have with chaos/complex systems is they are trying to understand systems written in a seemingly foreign language.

Distinguishing Between Chaos and Complex Systems Theories

There is a rapidly developing theory of chaos/complex systems, rooted primarily but not exclusively in mathematics and physics, and it is to that theoretical construct that we must adhere when we use any phrase that conjoins "complex" and "system." Technically, chaos theory is called *deterministic chaos*, to separate it from the everyday use of the term chaos—utter disorder and confusion. We refer to it simply as chaos theory. Both chaos and complex systems theories explore how the behavior of a system varies as the amount of energy/information it has to dissipate varies.

Chaos theory specifically studies why and how the behavior of simple systems—simple algorithms—becomes more complex and unpredictable as the energy/information the system dissipates increases. We use two definitions for chaos theory. The descriptive definition is "the quantitative study of unstable aperiodic behavior in deterministic non-linear systems." Frankly this is a bit opaque unless you already have a lot of experience observing the behavior of chaotic systems. The second definition is the behavior of the logistic system $X_{\text{next}} = rX(1-X)$, and since this system can be modelled in a computer, in class, in real time, it is the way we introduce chaos theory. Such a system has three behaviors, or attractor states. At low energies the system evolves to equilibrium (point attractor), at intermediate energies the system oscillates (limit cycle attractor), and at high energies the behavior turns chaotic meaning that although

the behavior is unpredictable, and may appear to be random noise, there are distinct patterns (strange attractors), properties, and behaviors. Clearly the same system—including even, for example, a pendulum—can move back and forth among the three attractor states, although the vast majority of natural systems behave as strange attractors.

Complex systems theory (or simply complexity theory) studies how systems with many “agents” that are already at high energy/information dissipation interact and behave. Complex systems possess virtually all the properties of chaos systems, which is why we study them first, but add their own properties and behaviors. The term “agent”, derived from artificial life studies, refers to the individual units that are interacting, like birds in a flock, sand grains in a ripple, or the individual units of friction along a fault zone.

What complexity theory demonstrates is that, by following simple rules, all the agents end up coordinating their behavior so that what emerges is not vernacular chaos, but deterministic chaos. Complex systems modelling is agent based—create n agents, assign each a few simple rules of behavior, and have them all interact with all simultaneously in a parallel processor. Intuitively it might seem the result would be random noise—in which case use statistical mechanics—but the actual result are evolutionary processes that lead to dynamically organized patterns. Experimentally, because the behavior of these systems is unpredictable, and we often want to explore how the behavior changes as various variables are tuned, computer simulation of the behavior with real time graphical output is a common strategy.

A concern sometimes expressed or implied is that if we adopt chaos/complex systems theory we must give up what we have observed or learned so far about natural systems. A biologist colleague recently said, “It appears to me that your definition of evolution allows one to skip over or ignore all the richness of the current theory of biological evolution” Nothing could be farther from the truth. By refracting what we already know about system behaviors through the mathematical prism of chaos/complex systems what we see is a new range of properties these systems possess, and gain a richer understanding of how many phenomena that now do not seem particularly related are related—because they all exhibit the same universality properties. Chaos/complex systems theory is not a replacement, not a substitute for the theories we already have, it is an enhancer—a multiplier—that make our observations and theories deeper, richer, subtler.

Teaching Chaos/Complex Systems Theory

We systematically introduce the three mechanisms of evolution, and chaos/complex systems theory concepts for understanding them in at least four courses we teach, although to different depths depending on the goals of the course. In all cases we first introduce the three mechanisms of evolution descriptively, giving a variety of common everyday examples. We then explore the behavior and properties of chaotic systems (about two 50

minute classes) and then use complexity theory models to talk about the different kinds of evolution (2-3 more 50 minute classes). The courses include a general education class dealing with the Earth and its environments (GGEOL 102: Environment: Earth), the historical geology class for majors (GEOL 230, Evolution of the Earth), another general education course dedicated to evolutionary systems of all kinds (GEOL 200, Evolutionary Systems), and, a new course in the environmental science minor (ENVT 200 - Environmental Systems Theory) (syllabi of the last two available at Fichter and Baedke, 2010). The concepts developed in the historical geology class are utilized in some of the upper level majors’ classes as appropriate; this latter is still a developing process since most of our courses were originally designed on equilibrium principles, and the transfer is taking thought, effort, and time. In all of the courses we demonstrate concepts with algorithms using computer based simulations where possible. In the evolutionary systems and environmental systems classes there are frequent breakouts to do computer lab experiments.

Chaos/complex systems concepts in the historical geology class are developed in one fell swoop near the beginning of the semester, while in the other courses concepts are introduced in steps throughout the semester, exactly how and in what depth depends on the subject matter and course level. In all cases we begin with one of the definitions of chaos—the logistic system, $X_{\text{next}} = rX(1-X)$ —which allows us to explicitly distinguish between equilibrium behaving systems, and non-equilibrium behaving systems and introduce concepts such as the computational viewpoint, positive/negative feedbacks, and the effects of ‘ r ’ values (rate of growth, or amount of energy and/or information the system has to dissipate) on the behavior of the systems (details below). The logistic system is then revisited over and over as we use it to explore ever deeper concepts, such as evolution to sensitive dependent critical states, avalanches of changes following power law distributions with fractal organization, and dynamic behavior as strange attractors that often exhibit bi-stable (hysteresis) behavior. Generally the approach of introducing the ideas in steps throughout the semester seems to be more effective pedagogically, since the periodic revisiting review and reinforce the ideas, but this requires that the course subject matter be organized to allow the progressive introduction of concepts.

Developing these ideas does takes class time (three to five 50 minute classes, depending on the number of examples or applications of each system we present), but probably no more than teaching other mathematical techniques or models. It would be better if our students came into our classes with a uniform understanding of these mathematical concepts, but, since they do not, then we must introduce them—because the fact is, complex systems theory is now being applied in progressively more areas. If we do not introduce our students to the basic language and techniques of chaos/complex systems theory they will not be prepared to think about and tackle these problems as professionals.

TABLE 1. SUMMARY OUTLINE OF THE MODELS, HOW THEY ARE REPRESENTED, AND THE LEARNING OUTCOMES IN THE ORDER THEY ARE DISCUSSED IN THE PAPER

Universality <i>Principles</i> of Chaos Theory		
Model	Representation	Principle
Logistic System - X_{next}	Time series diagrams	1. Computational viewpoint 2. Positive and negative feedback 3. r values 4. Deterministic predictable
Bifurcation Diagram	Part One: Generating the bifurcation diagram	5. Bifurcation = change in behavior 6. Instability increases with ‘ r ’
	Part Two: Self-Similarity (Fractals): Zooming in on the bifurcation cascade	7. Self similarity
	Fractal geometry (more depth on self similarity)	8. There is no typical or average size of events or objects. 9. Non-whole number dimensions
	Part Three: Feigenbaum ratios	10. All complex systems accelerate their rate of change at the same rate
	Part Four: Attenuating bifurcation diagram	11. All changes are preceded by increasing instability
Sensitive Dependence	X_{next} time series diagrams at 4.0000001 compared with 4.0000002	12. Minuscule changes in ‘ r ’ can result in dramatic changes in behavior
Power Laws	Log-log graph	13. Small-low energy-events are very common but do very little work. Large-high energy-events are very rare but do most of the work.
Strange Attractors	Phase space	14. Chaos/complex systems have behaviors that may superficially appear random, but have recognizable large scale patterns.
Principles of Elaborating Complex Evolutionary Systems		
<i>Typical Elaborating Evolutionary Models</i>		15. The general evolutionary algorithm – 1) differentiate, 2) select, 3) amplify, 4) repeat – is an extremely efficient and effective method of natural selection.
WordEvolv	Computer calculated algorithm	
John Muir Trail	Narrative description diagrams/charts	
Tierra	Narrative description with diagrams/charts	
Principles of <i>Self-Organizing</i> Complex Evolutionary Systems		
<i>Typical Self-Organizing Evolutionary Models</i>		16. Local Rules lead to Global Behavior, self organization arises spontaneously without design or purpose
Boids	MatFa’s Boids program (along with many other available programs)	
Self-Organized Criticality	Sand pile model	17. All natural open systems dissipating sufficient energy evolve-self-organize-to critical, sensitive dependent states which leads to avalanches of change that follow a power law distribution.
Cellular Automata	Life3000 program (along with many other available programs)	
Bak-Sneppen Ecosystem	Bak-Sneppen computer driven algorithm	18. In a complex system everything is connected with everything else. Nothing exists in isolation from the rest, sitting in a protected niche, independent and self-sufficient. 19. In a complex system no one can be completely safe, with complete control over their fate. Everyone is an innocent victim since there is no way one can fully protect oneself in such a world.

What follows are a series of instructional modules that include descriptions of the models, presentation notes, computer models where available, and anticipated learning outcomes. Each of these rubrics captures one of

the constructs that build to an understanding of chaos/complex systems. Note that we do not use all the models laid out in the rubric below in all the classes. In some classes clarity of fundamental concepts in the most concise

time frame possible consistent with how we plan to apply the concepts in the class is the goal. In other classes we use all these models and more. But, sometimes in a class where this is a first introduction to these unfamiliar ideas, one sharp, clear model that is easy to understand is better than many models showing variations on the same thing. After the ideas have had a chance to work in the unconscious for a while, the next time they are applied it is much more familiar. A summary of the learning objectives in the rubrics and how they are hierarchally related is shown in Table 1.

RUBRICS FOR TEACHING CHAOS THEORY

The human mind is built to think in terms of narratives, or stories. Chaos/complex systems theories are such narratives. They are not a series of unconnected or disconnected equations or models. If we present our students with a “complex system” but do not systematically develop the narrative that holds it together or makes sense of how all the pieces are interrelated, their ability to understand it will be hindered. Over the past 10 to 15 years we have experimented with and developed a variety of narratives or rubrics that systematically introduce the three mechanisms of evolution and chaos/complex systems. They are developed in a specific order to achieve specific ends of understanding. Each concept can be introduced qualitatively, followed by increasing mathematical or computational sophistication if desired; in most situations we keep it at the qualitative level because the ideas are visually apparent in the computer simulations. We do not mean this is the only possible narrative, but it is the one we have developed over years of experimenting with how to teach these ideas. We often include as many examples as we can, both inside and outside the geosciences, but this takes additional time. All these concepts, and the illustrations and animations we use, are contained in Power Point presentations. They can be downloaded at Fichter and Baedke (2010)

Logistic System: Time Series - Part One of Two Parts

Description: We always begin with the logistic system, $X_{\text{next}} = rX(1-X)$. It is a simple population growth model that produces an ‘S-shaped’ “logistic” curve representing early exponential growth followed by stabilization. This can be considered one definition of chaos theory, or rather the behavior of the system is one definition of chaos theory (Gleick, 1988). We usually spend an entire class on this model, and we are more deliberate with developing it than many of the ideas that follow (although if presented didactically it can take less time); likewise we will discuss this one more than the others.

In the logistic system populations always range between 0.0 (extinction) and 1.0 (largest conceivable population). To ground the model, it is introduced as a population growth model—we talk about Gypsy moth infestations—and how we might use the model to make predictions about future population sizes. The growth of the population—the positive feedback—is the rX . The reversal of population growth—the negative feedback—is the $(1-X)$. We also show a variety of logistic “S” curves garnered from the web to show how widely this model is

used in many studies. This demonstrates that the common, and not always correct understanding of the logistic system, is that it always does or should plot a diagnostic S-shaped curve.

Presentation: We do one run through of the calculations by hand at an initial population of $X=0.2$ and ‘ r ’ value of 2.7. The rest of the calculations are done with a program available at Fichter and Baedke (2010) which plots out the population sizes changes on a time series diagram (Figure 1). This is an interactive exercise between the students, the prof, and the model; i.e. we play a lot with the system, have fun with it, change parameters, experiment, debate with the students—just to see what happens. Initially, we run the model for 100 generations at 2.7 and ask the class to describe what they see; what happened. Next we ask for predictions of the behavior if we raise the ‘ r ’ value to 2.9, based on what they have just observed. This procedure is followed at increasing ‘ r ’ increments: 3.0, 3.1, 3.2 etc. until the system breaks at ‘ r ’ = < 4.1. The behavior evolves to amazing complexity. Figure 1 shows the range of behavior over a range of ‘ r ’ values. At 2.7 the system attenuates to a single population size—a point attractor—at 3.1 it oscillates between two population sizes out to seven decimal places, but at 3.0 the question becomes, “Does it ever stop attenuating?” The answer is, “No” at least not to 1 million generations, but after each experiment we keep asking, “Ok, based on what you have observed so far, what do you want to do next; make a hypothesis.” This is that sense of interactive play.

One of the other activities we have them do is to plot on a blank bifurcation diagram (see Figure 2; copy available at Fichter and Baedke, 2010) the final population sizes for each value of ‘ r ’ during the class discussion. It helps to make the bifurcation diagram explored next easier to grasp.

Anticipated Learning Outcomes:

1. **Computational viewpoint:** the idea that in a dynamic system the only way to know the outcome of an algorithm is to actually calculate it; there is no shorter route to knowing its behavior. This is not true at values below about 3.0, but becomes true at higher values. That this is not intuitively obvious is clear to the students because at higher ‘ r ’ values they are unable to predict the changing behavior of the system based on past behavior.
2. **Positive/negative feedback:** one central feature of all complex systems is that their behavior stems from the interplay of positive and negative feedbacks. This is a concept that students, when thrown into a natural complex system with many feedbacks operating, may have trouble grasping. The logistic system being so simple makes the influence of positive and negative feedback transparent.
3. **‘ r ’ values:** ‘ r ’ can be translated as rate of growth, although we use ‘ r ’ from this point on to talk about whether the ‘ r ’ value of any system is high (dissipating lots of energy and/or information), or

low (settling toward an equilibrium state).

4. **Deterministic does not equal predictable:** this is a very deep concept, especially if we explore its philosophical or theological roots (which in some classes we do). But, in classical science deterministic

equals predictable, and predictable means it is deterministic. The logistic system is undeniably deterministic, but at higher 'r' values all semblance of predictability breaks down. For understanding complex systems this is the most important concept gained from exploration of the logistic system.

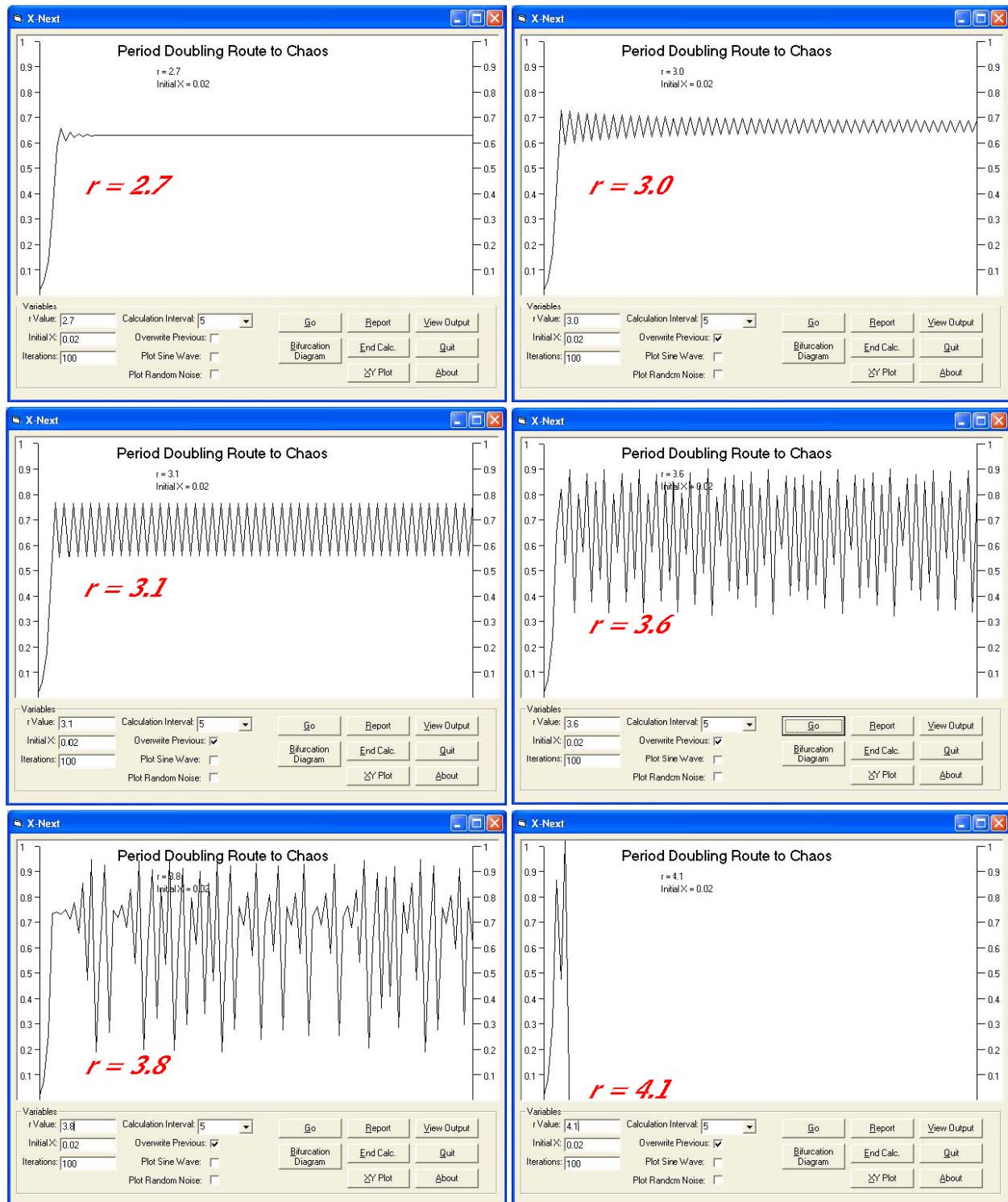


FIGURE 1. Series of 100 generation time-series diagrams generated by the logistic system at increasing values of 'r'. Note how the behavior becomes more energetic and complex as the 'r' values increase until the system self-destructs and goes extinct at an 'r' value a little over 4.0.

Bifurcation Diagram - Part One of Four Parts

Description: It is difficult to sense the behavior and properties of a system through many values of 'r' with a dozen or dozen and a half time series diagrams, so we convert them into a single, summative bifurcation diagram. The horizontal time series axis of the graph is replaced with 'r' values with the population sizes plotted at successively higher values of 'r'; the vertical axis remains population size. Figure 2 was created using the Xnextbif.exe program (available at Fichter and Baedke, 2010) to generate the data, which was then imported into Grapher to create the bifurcation diagram. Bifurcation is also called period doubling.

Presentation: A major point here is for students to understand different graphic representations, and in this case how the transformation is made from a time series diagram to a bifurcation diagram. This diagram is used multiple times to demonstrate other properties of chaos/complex systems. With Power Point animations (Fichter and Baedke, 2010) we make this transformation in class, and it takes 5 minutes or less.

Anticipated Learning Outcomes:

5. **Bifurcations are a change in the behavior of the system** and the entire range of behaviors can be summarized in one diagram. (One part of this is we want students to develop facility with different graphic representations.) Eric Newman, at his MyPhysicsLab web site has an applet of a chaotic pendulum. In just a couple of minutes we can demonstrate that bifurcations to complexity occur in a system that, going back to Galileo, is often taken as one of the best examples of a classical system.
6. **The harder a system is pushed, the higher the 'r' value, the more unstable and unpredictable its behavior becomes.** Unlike classical systems which can be described as simple, predictable, and with gradual change that goes to equilibrium, complex systems are ambiguous, unpredictable, and undergo sudden changes (bifurcations). Equilibrium means the system is dead.

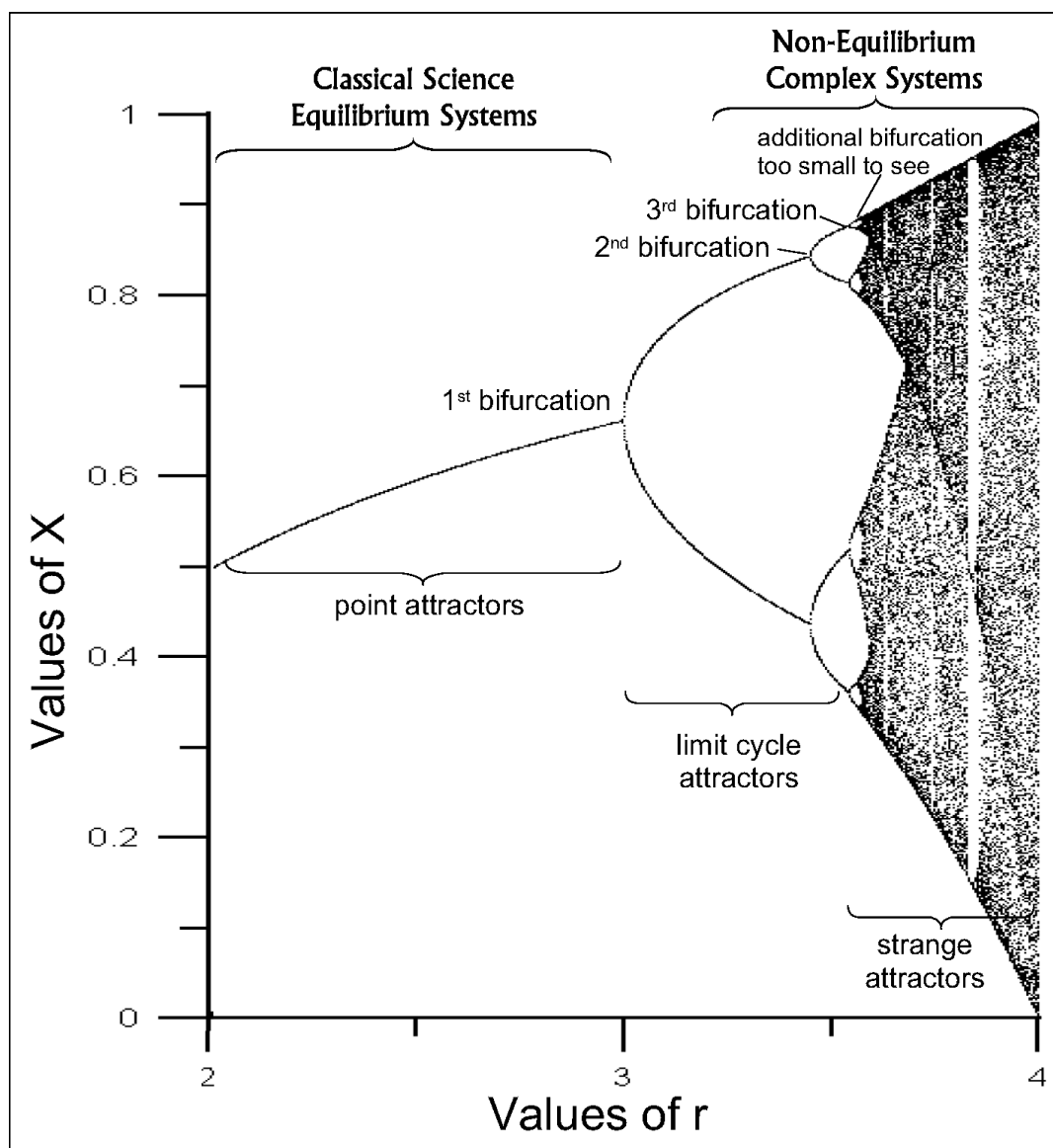


FIGURE 2. Bifurcation diagram for the logistic system. As the 'r' value increases the behavior becomes more complex, and more unpredictable. At 'r' values less than ~3.0 the system attenuates to a point attractor; that is, descends to equilibrium. At 'r' values between 3.0 and ~3.5 the system behaves as a limit cycle attractor, oscillating between 2, 4, 8, etc. population values. Above 'r' ~3.5 systems becomes deterministically chaotic, behaving as a strange attractor.

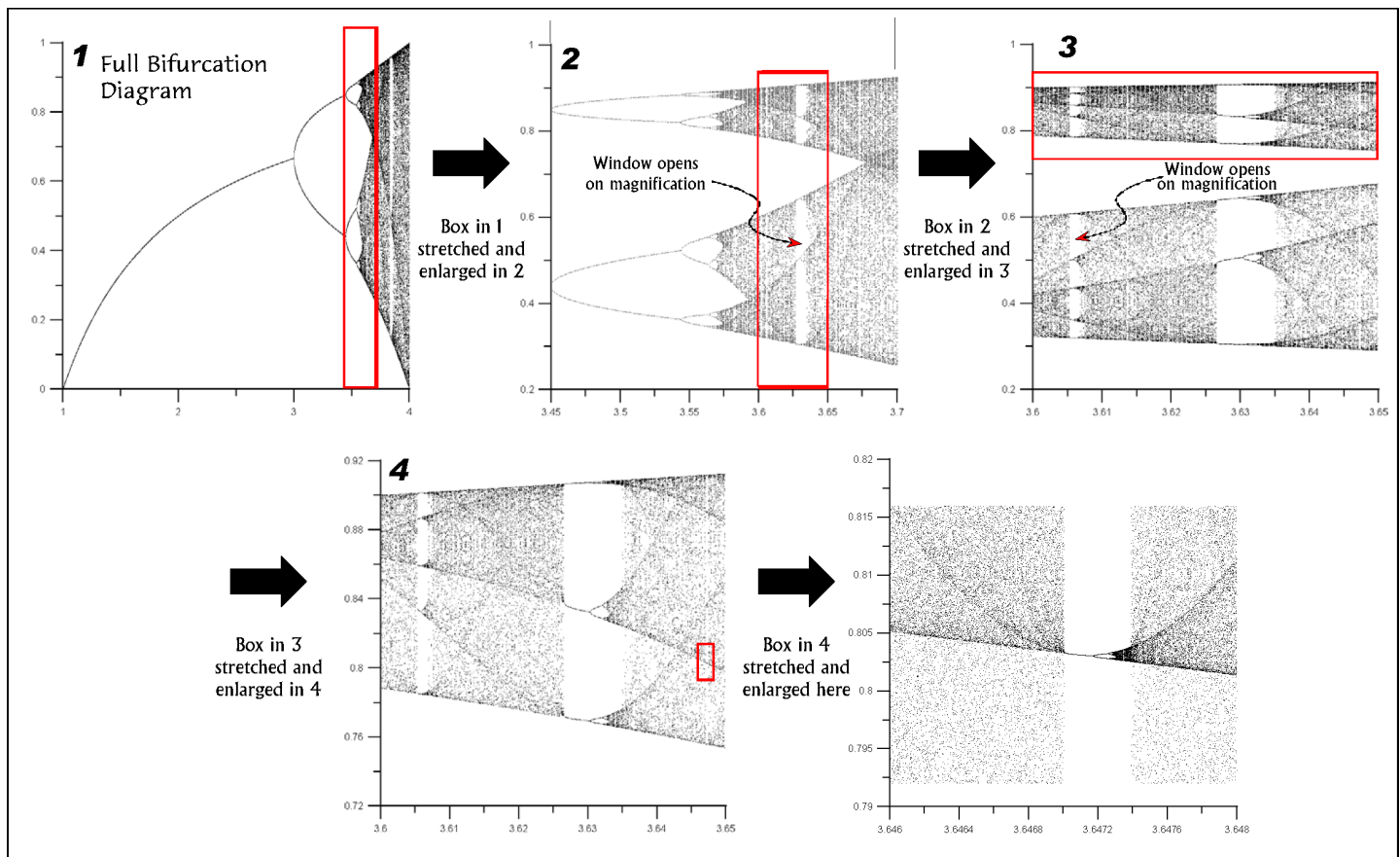


FIGURE 3. Series of zooms (magnifications) deeper and deeper into the bifurcation diagram demonstrating that the deeper one penetrates into the diagram, that is, the more decimal places the 'r' values are calculated to, the more detail is revealed. This is known as self-similarity, or as a fractal structure.

Self-Similarity (Fractals) - Bifurcation Diagram - Part Two of Four

Description: Virtually every object in the natural world has some self similarity properties, and they are important in understanding the structure of systems that evolve. Technically these are fractals, which is a whole branch of mathematics, but the basic idea of self-similarity can be quickly grasped visually with the bifurcation diagram.

Presentation: we illustrate self-similarity with a series of zooms (Figure 3) deeper and deeper into the bifurcation diagram, revealing the existence of smaller and smaller bifurcation cascades. We generate the data with the Xnextbif.exe program, and create the diagrams with Grapher (procedure available at Fichter and Baedke, 2010). In Power Point (see Fichter and Baedke, 2010) the presentation is simple, and effective. We also do a zoom cascade into the Mandelbrot set using an old program we have available (its web availability comes and goes, but search for mandelbrot at Cerious Software, Inc.), but there are web sites that do the same thing. These take about 10 minutes, and in some classes are enough. Sometimes it is necessary to explore self-similarity as fractal geometry, and we discuss that below.

Anticipated Learning Outcomes:

7. **Self-similarity is patterns, within patterns, within patterns**, so that you see complex detail at all scales of

observation, all generated by an iterative process. We show examples of river drainage systems, plant branching patterns, zoom in on the Dow Jones on the stock market index, look at Earth temperature patterns during the past ice age, sea level curves, cladograms, and examples of fractally generated landscapes.

Fractals (more depth on self similarity)

Description: In some classes we need a deeper understanding of fractals. Their close affinity with power-laws is important in understanding complex systems, and the behavior of natural systems. Our goal in introductory classes is not to present a treatise on fractal geometry, because it would be possible to spend hours of class time just getting started, and fractals are so fascinating, so ubiquitous, and so important that it is tempting to spend more and more time exploring them. Mainly what we want is for students to realize there is another geometry besides Euclidean, and that this geometry is how the natural world is constructed.

What students need to understand are the two signature properties of fractals; (1) fractal objects are generated by the iteration of a simple algorithm, that is, they develop by an evolutionary process, usually elaboration and self-organization. (which harkens them back to the logistic time series, which is generated by iteration), and (2) fractal objects have non-whole number

dimensions. We can introduce both of these ideas in about 15 minutes, enough to build on later if we need to. Showing a variety of examples to show fractal ubiquitousness is, at this stage, more important than the mathematics.

There are innumerable books and web sites that explore fractals, from basic to high mathematical sophistication. One very accessible book to the mathematics is Liebovitch (1998), while Briggs (1992) explores and illustrates the many ways fractals show up in nature and art.

Presentation:

Fractals are generated by an iterative process and we use the iconic Koch curve (or snowflake) as our class room demonstration (Figure 4). We have an animated Power Point (Fichter and Baedke, 2010) that illustrates this in less than 5 minutes. The algorithm is: 1) begin with a line, 2) divide the line into thirds, 3) remove the middle third, 4) fill the middle third space with two lines the length of the thirds, forming a triangle, repeat on each of the new lines. The Koch curve is so common that a search for "Koch curve" through Google/Images results in thousands of hits.

Fractals also have non-whole number dimensions. In mathematics the dimension of an object is calculated by $D = \log N / \log M$, where N is the number of new pieces

generated by an iteration (for the Koch curve from the initial 1 to 4), and M is the magnification that would be necessary to take each new piece generated by the iteration and enlarge it to the size of the original (for the Koch curve this is 3). Thus, the dimension of the Koch curve is $D = \log 4 / \log 3$ or .602/.477 which gives a fractal dimension of 1.26185. . . In some classes we use the same mathematics to demonstrate why one, two, and three dimensional Euclidean objects have those whole number dimensions.

There is so much more that can be done with fractals and their applications, and we sometimes do additional things, but at the introductory level this is where we stop, except when we have time to show a wide range of examples, both mathematical and natural.

Anticipated Learning Outcomes:

8. **There is no typical or average size of events or objects;** they come nested inside each other, patterns within patterns, within patterns. Peripheral to these discussions, but crucial when dealing with statistics, is that normal statistical analysis cannot be used on fractally organized data sets; there is no mean or standard deviation for fractal objects.
9. Unlike the geometry we are usually taught, most natural objects have **non-whole number dimensions**.

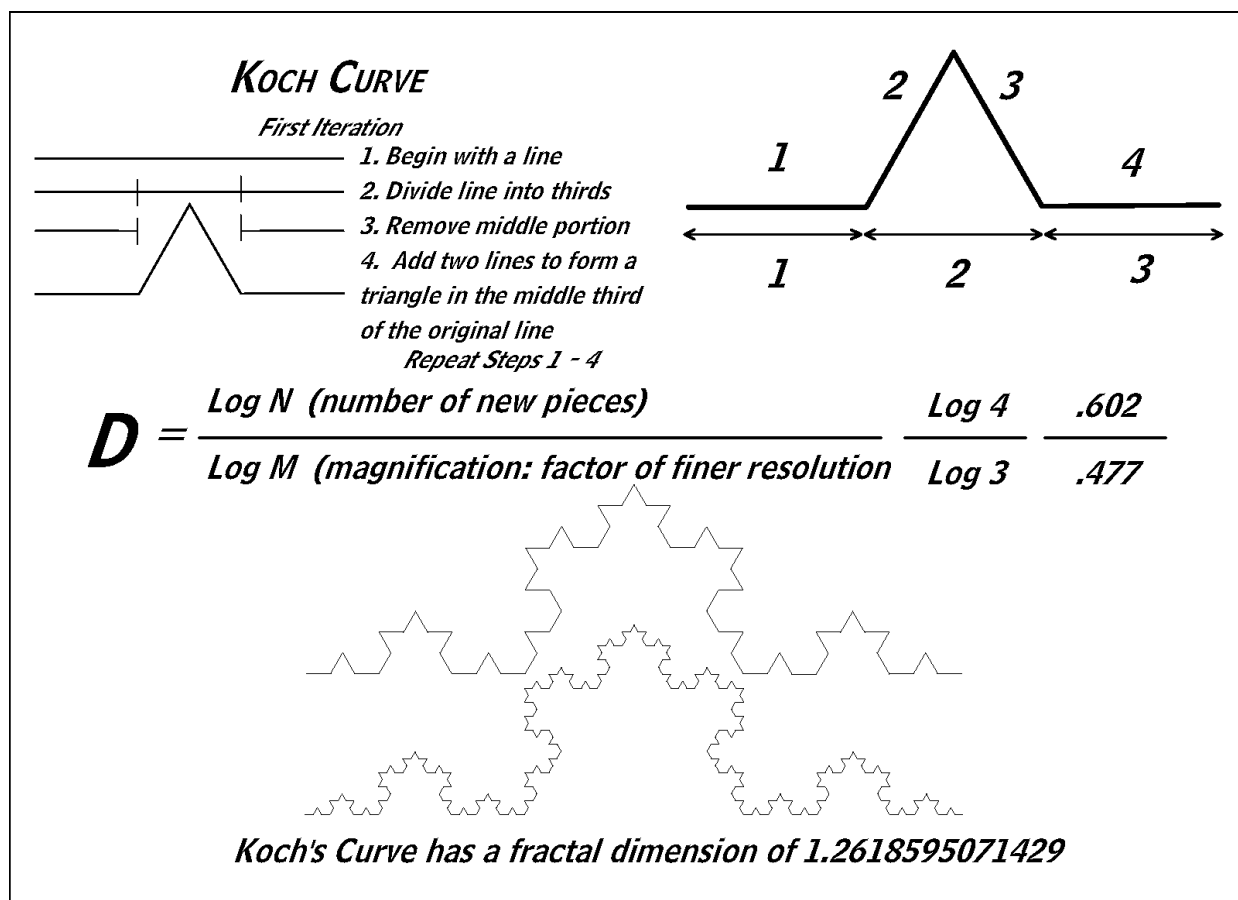


FIGURE 4. Fractal objects are generated by iteration of an algorithm, or formula. The Koch Curve is an example, generated by 4 steps, which are then repeated – iterated – over and over indefinitely, or as long as you want. To the right and below is the method of calculating the fractal dimension for the Koch curve.

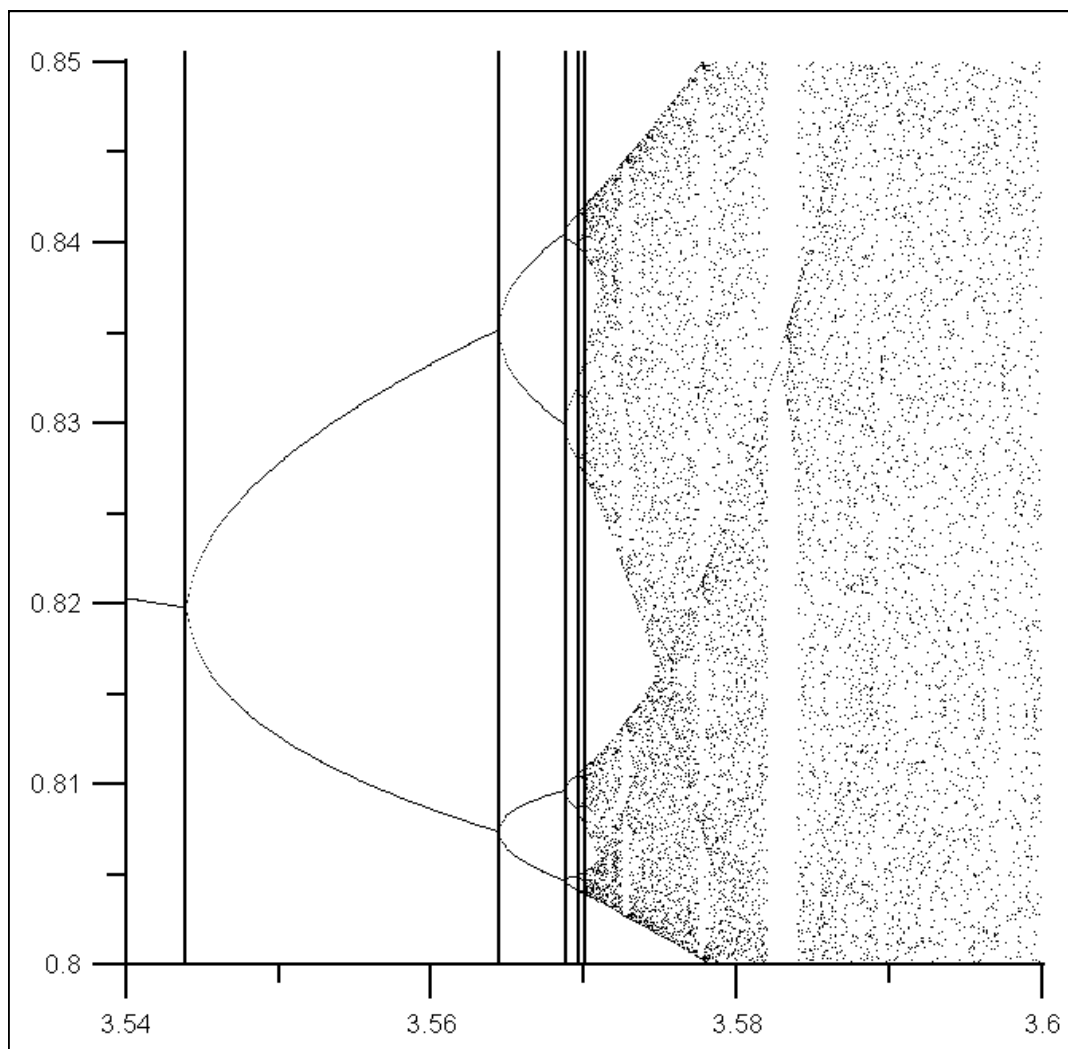


FIGURE 5. Bifurcation diagram showing that the rate of bifurcation accelerates at a constant rate with increasing 'r'. This is known as the Feigenbaum ratio. The period doubling ratio is given by the delta constant: 4.66921166091029 etc.

The faster change comes, the faster it comes - Bifurcation Diagram - Part Three of Four Parts

Description: Mathematically, Feigenbaum demonstrated that period doubling comes at a constant rate given by the delta constant: 4.669211... , and that this constant arises in any dynamical system that approaches chaotic behavior via period-doubling bifurcations: fluid-flow turbulence, electronic oscillators, chemical reactions, and even the Mandelbrot set.

Presentation: In introductory presentations we do not go into the mathematics of this, but while we have a bifurcation diagram up it is easy to demonstrate that each bifurcation comes faster than the previous one. In Power Point we just draw a series of vertical lines at each bifurcation and note that they come faster and faster (Figure 5).

Anticipated Learning Outcomes:

10. **All complex systems accelerate their rate of change – bifurcations – at the same rate.** This is a universality property; if this rate of change in behavior of a system is detected, the system is a complex system.

Change is always accompanied by increasing instability - Bifurcation Diagram - Part Four of four

Description: People often express surprise when a system undergoes a sudden change. The 2008-2009 crash in the stock market is a recent example. Up to a few weeks before the system collapsed economists were saying the fundamentals were strong and the economy was doing just fine, and then it crashed. (Reinhart and Ragoff, 2009, demonstrate that the belief that economic change does not occur suddenly and unexpectedly has been true for at least the past 8 centuries.) Similarly, until about a decade ago we generally believed climate change was gradual – taking thousands of years – until ice core data demonstrated climate regimes can shift in less than a decade. On the other hand, people who understand the behavior of complex systems are not surprised by rapid changes. For example, a decade or two before the market crash, swings in the Dow Jones Industrial Average kept getting larger and larger. Often in real time we can detect when a system is approaching a bifurcation; its 'r' value is climbing and behavior begins to become more erratic.

Presentation: Figure 6 is a detail of the first and second bifurcations in the logistic system, but instead of plotting

only the last few population values, after the system had settled down as much as it was going to, we plotted a wider range of population values on the way to settling down. The result is, visually we can see that as a system approaches a bifurcation it goes from relative stability, to increasing swings, to splitting, to settling down in the new state. The concept can be grasped visually, and takes about 5 minutes to present.

Anticipated Learning Outcomes:

11. **All changes – bifurcations – in a system are preceded by increasing instability in the behavior of the system.** After splitting the system settles down toward stability again.

Sensitive Dependence - The Butterfly Effect - Logistic Time Series - Part Two

Description: When we ask students if they are familiar with the butterfly effect, many say they have never heard of it. After some discussion of, for example, the aphorism “The straw that broke the camel’s back,” or the poem that begins “For want of a nail the shoe was lost . . .”, or the movies “The Butterfly Effect” and “Sliding Doors” most will begin to nod knowingly. But, this concept must be sourced back to chaos/complex systems behavior.

Presentation: One simple and quick way to illustrate sensitive dependence is to run the logistic program first at an ‘r’ value of 4.000001, and then open and run another version at 4.000002. In Power Point we animate a line that

traces the trajectory on the time series at 4.000001, and then move it to the adjacent time series diagram at 4.000002. It is easy to see the divergence in the trajectories (available at Fichter and Baedke, 2010). This is quick and easy, but we often go on to spend a little time discussing Edward Lorenz and his discovery of the phenomena (although the mathematician Poincare recognized it around the beginning of the 20th century.)

Anticipated Learning Outcome:

12. **In a complex system at high ‘r’ values a difference as little as one part in a million can result in different histories for the system.**

Power Law Relationships

Description: A power law relationship exists when the events in a complex system plot as a straight line with a negative slope on a log-log graph (see for example, Barton, 2001), or if that data plots as an asymptote on a linear graph. Power law relationships are a signature feature of complex systems.

Presentation: Most students are probably familiar with log-log graphs, but not by the name “power law relationship.” We do a short introduction to the mathematics of power laws, both on a linear graph and a log-log graph, and then give several examples, including stock market prices, earthquake distributions, flood/storm/hurricane intensities, extinction sizes, and Zipf’s law as it applies to word usage, the size of cities, and

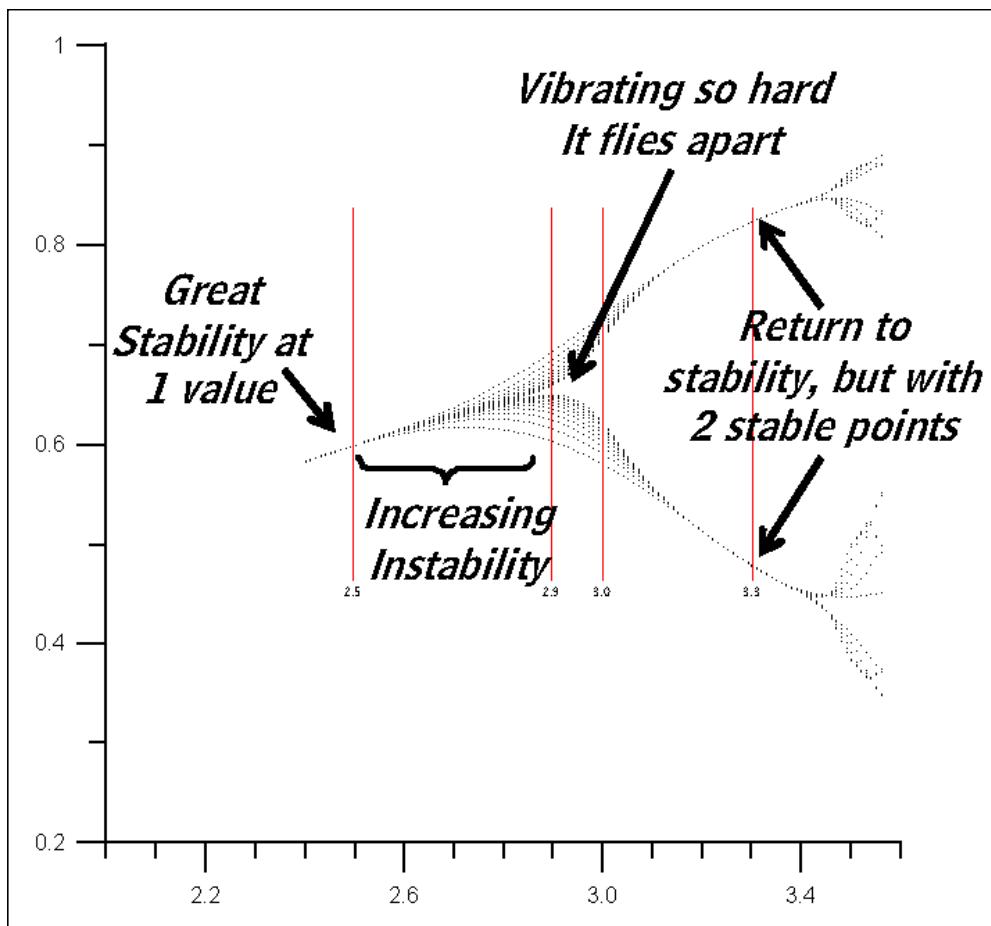


FIGURE 6. A detail of the first and second bifurcations in the logistic system, where for each value of ‘r’ we plotted not just the final population values but also the attenuating population values. This shows that as the system approaches a bifurcation it becomes increasingly unstable until it is finally vibrating hard enough to bifurcate, after which it settles down again to stability at two values. This pattern is true for all bifurcations.

music structures. If we go into greater depth it is to demonstrate the relationship between fractal geometry and power law distributions.

The deeper significance of power laws is that they are generated by—are the result of—an evolutionary process. This is best demonstrated in self-organizing evolutionary systems, such as self-organized criticality or cellular automata, but shows up in all evolutionary systems.

Anticipated Learning Outcome:

13. **Small—low energy—events are very common, but do very little work. Large—high energy—events are very rare, but do most of the work in a system.** Once we have established this concept we use in many different situations. In addition to the examples listed above the senior author uses power laws in a stratigraphy course to discuss preservation potential and time representation of events in the stratigraphic record.

Strange Attractors

Description: In mathematics an attractor is a region of phase space that "attracts" all nearby points as time passes. A phase space is simply an XY or XYZ graph (although it could be at higher dimensions) with grid variables such as position and velocity (but not time). The changing values of the system have a trajectory which moves across the phase space through time toward the attractor, like a ball rolling down a hilly landscape toward the valley it is attracted to. Each dynamical system and each 'r' value generates a particular type of attractor.

There are three basic kinds of attractors. A point attractor is generated when a low energy system decays to equilibrium, like the diminishing swings of a pendulum to motionlessness. In phase space this appears as a single point (or a spiral down to a single point). In the logistic system time series it appears as the population attenuating to one value (Figure 2). A limit cycle attractor repeats the exact same trajectory in phase space, like a clock pendulum driven by some constant energy source (although in fact clock pendulums always exhibit some strange attractor irregularity, sometimes called pink or 1/f noise.) In the logistic system the limit cycle attractor is an exact oscillation among two or more population sizes, out to the 7 decimal places the program shows (Figure 2) (e.g. 'r' value = 3.1). A strange attractor is one which in phase space has a recognizable shape, but never exactly repeats its trajectory. The Lorenz attractor is the best known (an internet search for images produces thousands of examples.) Strange attractors are the iconic behavior of chaos/complex systems.

In a classical framework—a system that descends directly to equilibrium—the only attractor is the limit cycle; such systems are non-evolutionary, and we are not interested in them in this discourse. Of course, we model some systems as limit cycles, such as a driven pendulum, but that can only be sustained by the continuous input of moderate amounts of energy. These are also non-evolutionary but we are moderately interested in these because, with just a little more energy, they will transform into evolutionary strange attractors that pass through a

series of bifurcations as energy increases. But, as we see with the logistic system, if the energy dissipation level continues to rise, the system undergoes a series of accelerating bifurcations until we see a non-repeating pattern, which plots in phase space as a strange attractor. From an evolutionary viewpoint, only the strange attractor is dissipating enough energy—is unstable enough—to be able shift—evolve—into an alternative pattern.

Presentation: We use a computer program (available at Fichter and Baedke, 2010) that shows evolution of the Lorenz strange attractor, although Cross (2009) has a nice applet of the system with suggestions for experimenting with the system on line.

Anticipated Learning Outcome:

14. **Complex systems have behaviors that may superficially appear random, but have recognizable larger scale patterns.** The weather is an example. Forecasters have trouble predicting the exact weather 3 days out, but the basic pattern of moving high and low pressure systems is universal. Likewise, this spring may not have been like the last spring, but the seasons follow in the same order every time.

RUBRICS FOR TEACHING EVOLVING COMPLEX SYSTEMS

As scientists and educators we are at the beginning of understanding complex systems and their evolution, not the end. It has taken us centuries to build the equilibrium concepts that underlie most ideas in science today. We have decades of work in front of us to flesh out evolution as a complex system, although physics and chemistry at their cutting edges have progressed farther along this path. Mathematicians have done much work applying chaos and complex systems theory to biological systems (e.g. Artificial Life studies), but biology as a discipline still has a long way to go to rethink Neo-Darwinian principles as a complex system. On the other hand, with a few exceptions such as earthquake systems, the geosciences are farthest away from modelling change as evolutionary complex systems.

There are two basic approaches to demonstrating that evolutionary systems are complex systems, and they are not mutually exclusive. The first is the tried and true method of building mathematical models that reproduce the behaviors of real systems. However, complex system models must incorporate non-linearity, are typically agent-based, and must be computational. Computational means an algorithm is established that possesses various rules—which typically can be tuned for experimental purposes—and then allowed to run with the output being graphed in phase space, a bifurcation diagram, or some similar way. Because the outcomes of these models often behave as a strange attractor their evaluations are often first qualitative, although numerical outcomes—such as power-law relationships, fractal structure, sensitive dependence, avalanche behavior (explored in more detail below)—are also sought. These models follow the computational viewpoint; the results are difficult or impossible to

predicted ahead of time; we just have to run the system and see what happens.

The second approach for recognizing complex evolutionary systems is to compare data from real systems to see if they possess the same properties as generated by theoretical chaos/complex systems models. One seminal idea of universality is that if a system can be shown to possess one of the universality properties it also possesses other universality properties, and the presence of these properties makes the system a complex system. Below we explain in further detail how we use some of these elaborating and self-organizing models in our classes to demonstrate complex evolutionary mechanisms. We are confined to elaborating and self-organizing evolution since we are unaware of transparent computer models that allow complex modelling and experimentation with fractionation as a complex evolutionary system.

Depending on the goals of the course and what we are trying to accomplish, we discuss, demonstrate in class, or do lab experiments with different combinations of these models. Most of the models are too elaborate to take apart and describe here, but a variety of suggestions and resources are provided at Fichter and Baedke, 2010.

Typical Elaborating Evolutionary Models

Both biological and non-biological elaborating evolution are usually explored with the use of genetic algorithms. That is, we create a bunch of electronic “ants” with certain genetic codes (stored in the computer memory), with powers of biological reproduction (mutations, cross overs, inversions, natural selection), place them in an electronic ecosystem that possesses various environmental parameters that results in selection, and let it run. A wide spectrum of these have been created from very simple to very sophisticated. The best introduction to this subject is Levy (1993). A few we commonly use are described below.

WordEvolv

Description: WordEvolv is a simple, powerful, and very effective demonstration of how efficiently a mutation/natural selection strategy can evolve a meaningful pattern from a meaningless string of random letters. This is also a non-biological demonstration of the General Evolutionary Algorithm (see above) since this is clearly an elaborating evolutionary system, but we are not working with direct biological analogs.

Presentation: copy of the program is available at Fichter and Baedke (2010). Usually this is a class demonstration, but depending on the class we may have them run some experiments themselves in the lab. We begin by establishing a target string; a phrase, or someone’s name. This becomes the fitness function. Next the program calculates how long it would take to generate the target string if we were doing it completely at random. For the target string, “What is this phrase” there are 1.2×10^{17} possible strings, which on my computer would take years to generate. However, beginning with a random string the same length as the target string, and running it through a mutation and selection process (three different

strategies are available in the program) the target string can be evolved in generally between 50 and 75 generations (depending on the random starting string). A “view text file” allows everyone to see the evolutionary steps from the initial random string to the final string.

Anticipated Learning Outcome:

15. The **general evolutionary algorithm**—1) differentiate, 2) select, 3) amplify, 4) repeat—is an extremely efficient and effective method of natural selection.

John Muir Trail: Another genetic algorithm is the John Muir Trail developed by an MIT research group to discover how efficiently a “species” of ants can learn to run a trail. It is described by Levy (1993) and Johnson (2002). We do not have a program to run this, but we describe its workings and outcomes in class (power point at Fichter and Baedke, 2010).

Tierra. One of the most advanced genetic algorithms is Tom Ray’s Tierra. Tierra begins with one simple string of machine code that can do only one thing—create a copy of itself. Then, with a low level of background mutation, and millions of generations new species are generated, ecosystems created, parasites evolve, which are then overtaken by other hyper-parasites, which are then overtaken by hyper-hyper parasites. What we get is what we observe in nature, except that with a genetic algorithm we get a complete data set of all the evolutionary changes taking place, available for analysis. One of the things such systems demonstrate is that many of these genetic algorithms undergo punctuational evolutionary change, and have waves of extinctions that follow a power law (similar to the extinction data of Raup and Sepkoski, 1986; see Fichter, Pyle, Whitmeyer, 2010 for a discussion of extinction as a complex system and seminal references). All these point to elaborating evolution as a complex system.

Self-organizing Evolutionary Models

Self-organization—creating order out of disorder without intelligent design, or selection—is not intuitively obvious. Indeed, as the intelligent design debate shows, it is counter intuitive. Self-organization is not easy to reason out—it is not a deductive process. It is inductive; it must be experienced—hence the computational viewpoint. And, it is not like a genetic algorithm—e.g. WordEvolv—although self-organization does result from the application of simple rules. Yet, self-organizing evolutionary processes are probably more pervasive and more important than elaborating and fractionating evolutionary mechanisms (see Ball, 2001); even biology is pervaded by self-organizing evolutionary processes (e.g. Goodwin, 2001).

At the introductory level the foremost issue is coming to accept that self-organization occurs, and that it is reasonable, in spite of the second law of thermodynamics and entropy. John Holland in *Hidden Order* (1996) talking about how mercantile systems self-organize—i.e. Adam Smith’s “invisible hand”—says that, intuitively, the process seems a sort of magic that is everywhere taken for

granted. Therefore, at the introductory level there is a level of logical disbelief that must be addressed, and we focus on several systems where self-organization can be experienced.

We use four self-organizing systems in varying combination, sophistication, and depth of analysis to explore self-organization, depending on the goals of the class. Although superficially they may seem to not have a lot in common they all boil down to the acting out of local rules by each agent which leads to global behavior (“local rules/global behavior.”) The four systems are boids, self-organized criticality, cellular automata, and the Bak-Sneppen ecosystem.

Boids

Description: Boids (“bird-like object”) were developed by Craig Reynolds (2001) in the mid 1980s to demonstrate that the flocking behavior of birds could be explained by the action of simple rules being followed by each boid. The rules were 1) steer toward the center of the flock, 2) do not get too close to flock mates, 3) steer toward the average heading of the flock mates. Beginning with an initial random positioning and movement of the boids, and each boid following the local rules for its behavior, the outcome is flocking behavior—the boids will end up tracking together. A flock will even split and fly around an object placed in its path and rejoin on the opposite side.

Presentation: We use MatFa’s boids program (available at Fichter and Baedke, 2010), but there are many out there for download (e.g. Reynolds, 2001). MatFa’s program has two nice features (in addition to everything else it does), a “turmoil button” which randomly scatters the boids with a click, and an obstacle creator the boids will then fly around. This demonstration can be done in a couple of minutes, but it is effective at demonstrating self-organizing flocking behavior from local rules, and without the use of higher intelligence functions. We emphasize the local rules/global behavior concept, but do not belabour the rules in introductory courses; this is to be experienced, not so much analysed. On the other hand, we extend the demonstration into a discussion of traffic behavior, and walking down a crowded sidewalk without bumping into people. This is so intuitive that students immediately grasp it. This is also the principle behind Adam Smith’s “invisible hand”, sometimes restated as private virtue (each individual doing what is best for themselves) leads to public virtue (wealth in the society increases).

Anticipated Learning Outcome:

16. **Local Rules lead to Global Behavior**, self-organization arises spontaneously without design, or purpose, or teleological mechanisms.

Self-organized Criticality (SOC)

Description: Per Bak (1999) developed self organization as an evolutionary system by modelling sand pile behavior. Being a physicist, though, instead of starting with real sand piles, he built a computer model of sand pile behavior. Later workers then tested his ideas out by studying real sand pile behaviour (there are complications

which Bak discusses). This is one of the most important models we introduce, and it is done in every class where we talk about these ideas. The concepts are useful in numerous direct and indirect circumstances. The best way to get into this is to read Bak’s (1999) book, but also look at Jensen, (1998; accessible with basic math development) and Hergarten (2002; very high level mathematics). Bak has worked to apply SOC to a diversity of natural systems, both biological and physical. His SOC model of earthquake activity along fault zones is the most widespread example in the geosciences, but SOC has innumerable other applications.

Presentation: We project a picture from Bak’s book of a hand dribbling sand to build a sand pile, while telling this story, using our hands to conjure up images “Now, imagine a platform, perfectly flat, and a container above it containing sand, with a valve that allows us to dribble sand one grain at a time if we wish...and so on” Done this way the demonstration is rivetting to a class as we talk through building the sand pile from single grains, to a low mound, to an ever steepening pile that evolves to be more and more critical, adjusting itself through avalanches that reduce the criticality, while additional sand builds the criticality back up again, leading to another avalanche. The term avalanche—a sudden release of energy—refers naturally to sand pile behavior, but we use “avalanche” to refer to any abrupt shift in state or sudden release of energy in any self-organizing critical system.

Anticipated Learning Outcome:

17. **All natural open systems dissipating sufficient energy evolve—self-organize—to critical, sensitive dependent states which lead to avalanches of change that follow a power law distribution.**

Cellular Automata (CA)

Description: A cellular automata is a grid of cells, something like a checkerboard. Each cell in the grid may take on a series of states depending on the rules of the system; simplistically, the cells are either alive (light up), or dead (blank), although they can be any set of states a person desires, like the three primary colors, or the six colors of the secondary color wheel. Cellular automata are excellent systems to visually demonstrate self-organization, but take a little more time to develop than boids. On the other hand, CA have so many applications in so many disciplines for so many different kinds of complex systems that they are becoming *de rigeur* in some fields.

The simplest CA local rules are “survival neighbors,” and “birth neighbors.” Take a live cell, what will insure its survival to the next generation? The live cell is surrounded by eight other cells. It will survive if at least 2 or 3 of the surrounding cells are alive, but not more than 3; any 2 or 3 of the 8 cells will do. Take a dead cell, what will allow it to birth the next generation? It will come alive if 3 and only 3, any 3, of the surrounding cells are also alive. A plethora different rule sets are possible, and for some systems, such as oscillating chemical reactions, we want rules that model the chemical processes. At the

introductory level we primarily want to demonstrate self-organization.

Presentation: We use one of the oldest CA programs called Life3000 by David Bunnell (see Fichter and Baedke, 2010 for a copy). It is simple, neat, and very easy to use. Since we are demonstrating self-organization we take the time in class to discuss the local rules and have the students, via discussion, calculate the fate of a single cell, or variety of cell combinations. We want them to unambiguously understand that the outcome of the demonstrations is not pre-programmed in; that it is deterministic and stems solely from the local rules/global behavior concept.

The demonstration itself is simple. Hold the mouse button down while swirling the cursor across the grid to create an unorganized splay of live cells. Then run the program; the cells, after varying times and complexities of activity, will settle down to an end state of simple, static, geometric arrangements of cells (point attractors), or oscillating arrangements of cells (limit cycle attractors). Run the experiment as many times as desired, each time starting off with a different random splay of cells. The system will always self-organize to the same basic kinds of arrangements, although the distribution will be different for each different initial state. Sensitive dependence can also be demonstrated: after the system has stabilized; add one live cell and set it running again. Depending on where the cell is located there will occur another avalanche of changes lasting anywhere from 1 generation, to many dozens of generations. If one continues to do this, keeping track of the length of the avalanches, they will be seen to follow a power law distribution, a signature property of complex systems.

If we have the time we do demonstrations from some of the other CA programs available on line to show the complexity and intricacy of behaviors that occur with different rule sets (search for Mirek's Celebration on line; if still available it has a large library of demonstrations, and examples of a wide variety of rule sets. Look at the 'Must see' folder.) There is almost no limit to the modelling possible with CA; it is a big field in mathematics. See for example Forrest and Haff (1992) for a CA model on eolian ripple formation.

Anticipated Learning Outcome (same as last, but with a different model):

17. **All natural open systems dissipating sufficient energy evolve—self-organize—to critical, sensitive dependent states which lead to avalanches of change that follow a power law distribution.**

Bak-Sneppen Ecosystem

Description: Bak-Sneppen is an ecosystem used to model the co-evolution between interacting species following two simple rules. The technical paper is Bak and Sneppen (1993), but Bak also discusses it in his book "How Nature Works" (1996).

In the Bak-Sneppen ecosystem we create n species each with a fitness chosen at random between 0.0 and 1.0 (maximum fitness). When the model is running the fitness

of each species changes because of its relationships with other species following two simple rules. Rule One: find the one species with the lowest fitness and randomly change its fitness. Rule Two: at the same time the lowest fit species is changed, also change the fitness—at random—of the species to the immediate left and right. Repeat next generation.

The entire ecosystem has a threshold fitness, determined in the first generation by the fitness of the lowest fitness species in the ecosystem. After that, threshold fitness rises in any generation in which the fitness of the lowest fit species rises above the previous threshold fitness. An avalanche is the number of generations between rises in the threshold fitness. During an avalanche lots of species will develop fitnesses below the threshold, but the threshold will rise only when the lowest of these fitnesses happen, by chance, to rise above the current threshold. While running, the avalanches appear as a zone about 10-15 species wide undergoing fitness changes that sweeps back and forth across the ecosystem.

Presentation: A copy of the Bak-Sneppen model is available at Fichter and Baedke (2010). There is also available a Power Point presentation that shows how we use the program, documentation and help files for the program, the kinds of questions we explore, and some of the universality property outcomes from Bak's publications. There are many lessons to be gleaned from demonstrating the Bak-Sneppen model, and this is one of the models we consistently use in most classes.

From the self-organizing perspective, the important question is, "Can any organized outcome evolve when the initial state is set at random, and all fitness changes occur at random?" In fact, the system self-organizes via a punctuated equilibria strategy, and results in extinctions (any time the fitness of a species changes for what ever reason) that follow power-law distributions. The sweeping back and forth pattern of the avalanches is also fractal.

Anticipated Learning Outcome (in addition to self-organization and the universality properties); with an ecosystem/environmental systems bent:

18. In a complex system everything is connected with everything else. Nothing exists in isolation from the rest, sitting in a protected niche, independent and self-sufficient.
19. In a complex system no one can be completely safe, with complete control over their fate. Everyone has the potential to be an innocent victim since there is no way one can fully protect oneself from external actions.

Fractionating Evolutionary Models

We are unaware of any computer-based models that demonstrate, or allow one to experiment with, fractionating evolution, in the same sense that we can experiment with genetic algorithms or self-organizing systems. (One exception are oscillating systems. We

introduce these as self-organizing systems, but a common outcome is fractionation. This, however, is a higher level analysis than these rubrics explore.) This is unfortunate since in the geosciences fractionating evolution is so important. Nonetheless, we develop many fractionating systems as overtly evolutionary systems in our geosciences courses, and teach them as evolutionary narratives, from the fractionation of the nebular gas and dust cloud, to the stratification of the early Earth, to igneous rock fractionating evolution, sedimentary rock weathering and fractionating evolution, and even the fractionation of oxygen isotopes in temperature studies of ice cores. Since all of these result in increases in complexity, diversity, order, and/or interconnectedness they are properly studied as evolutionary systems. We have just not been able yet to put them clearly into agent-based complex system models based on physical and chemical principles in the same way we have been able to develop self-organizing evolutionary models.

Further Ideas

The strategies and rubrics developed above are only an introduction to complex systems thinking and modeling. But, depending on your goals, the learning objective can be applied to an understanding of many other complex systems, such as oscillating chemical reactions (reaction-diffusion and activator-inhibitor systems), hysteresis (bistable) systems, network theory, and autocatalytic networks. We use these and other complex systems concepts in various classes to talk about the origin of life, ecosystem organization, game theory, extinction events, rise and collapse of complex cultures, and a variety of other systems. The applications are almost endless.

ASSESSMENT OF STUDENT ACCEPTANCE OF CHAOS/COMPLEX SYSTEMS THINKING

With the introduction of any educational innovation, the central question is whether the innovation is any more effective at helping students reach learning goals than the methods currently employed. To document this effectiveness, one needs to consider the collection of assessment data in the service of evaluation.

Assessment involves the determination of the level to which students have actually met learning expectations, as determined by the use of valid and reliable instrumentation (Kizlik, 2009; Pyle, 2010). These data are then used to inform decisions on the effectiveness of instructional strategies and make accurate statements on the "value-addedness" of these strategies to students' overall education (Pyle, 2010). Data collected typically consists of cognitive information, but as the approach described in this manuscript is as much about impacting how students think about the natural world as it is about what they know about systems, it is vital that changes in students' dispositions or habits of mind be documented.

In two of the four courses where chaos/complex systems concepts are systematically taught (GEOL 200, ENVT 200), students' perceptions of their understanding and appreciation of chaos/complex systems before they took the course, and after are assessed. Students in these

courses are typically non-science majors, making up more than half of the enrollment. Because both these courses incorporate computer-based experiments, class size is limited by the size of the computer lab, usually 20-22 students. In addition to the technical aspects developed by the rubrics described above, we spend time in both classes putting chaos/complex systems theories into historical (development of science) and philosophical context, for a couple of reasons. The assumption is made that all students entering the course have been inculcated in classical science concepts throughout their education. When asked, the vast majority of students have indicated that they have had little or no exposure to chaos/complex system principles. The historical/philosophical context is also important because the learning objectives go beyond technical expertise, but also include generating a willingness to use different strategies for tackling problems to produce a range of solutions. We want students to learn to evaluate each kind of problem and the kinds of thinking strategies that are likely to yield insight and understanding. Our expectation is that in the process of learning through the rubrics presented here the students will develop a new gestalt, and shift to a more encompassing frame of reference. These goals are present in all four courses, but are assessed in only two.

We currently are developing a cognitive assessment tool for student learning of chaos/complex systems principles and applications. It is desirable to have an instrument that can be used in all four courses, based on knowledge surveys (Nuhfer, 2008). (A knowledge survey consists of course learning objectives framed as questions asked at the beginning and end of the course. Students do not provide actual answers, but instead respond by a three-point rating of their confidence to respond with competence to each query. Because students do not provide actual answers the survey can cover an entire course's content in depth, and the before and after comparisons give insight into the effectiveness of various parts of the course.)

Anecdotally, we have observed over many experiences teaching these subjects, using the rubrics presented in this paper to frame the instruction, that students have no more difficulty learning the technical concepts of chaos/complex systems than students have learning any other new subject. More important, students taught with these rubrics have shown a ready facility to recognize and understand complex systems principles in the analysis of real world applications. This is revealed when classically understood systems are re-examined during class discussions/Socratic seminars in terms of the universality principles of chaos/complex systems. We are satisfied that instruction following these rubrics have guided the majority of students in the application of chaos/complex systems concepts with relatively little difficulty. It is also clear that they understand much about the limits of classical methods and chaos/complex systems methods.

Because students enter these classes knowing virtually nothing about chaos/complex systems theories we might expect some of them to be skeptical of or to reject these ideas. Statements such as: "This goes against everything I

have been taught in my other science classes about how natural systems work," or "I cannot accept this because my church says evolution can't be true" reflect dispositions that could obstruct learning. Colleagues imbued in classical concepts have offered the same skepticism, simply dismissing chaos/complex systems ideas. Knowing that skepticism exists, the rubric progression is designed to develop the chaos/complex systems concepts non-confrontationally, logically, sequentially, and dispassionately (except for the wonder of watching a bifurcation diagram unfold on the screen, or a fractal object emerge from an algorithm.) Experience has shown that presenting chaos/complex systems principles in a disjointed or fragmentary way usually leads to misunderstanding and confusion, followed by skepticism, if not rejection. Just as the principles of algebra must be developed systematically, so must chaos/complex systems principles.

It is important, therefore, to determine students' dispositions by assessing how receptive they are to the ideas, how willing they are to continue to learn about them, apply them, and be influenced by them. If their interest ends with the end of the course, then the larger course goals go unmet. These assessments collect both qualitative and quantitative information, but only the quantitative results that directly address students' acceptance of chaos/complex systems strategies are reported here.

At the end of the semester students are asked, "How much do you feel you grew in each of these areas from the beginning to the end of the semester." A sample question format is below.

Your knowledge and understanding of chaos and complex systems theories and their application to environmental problems.													
Class Start							Change During Class						
Little							A lot						
1	2	3	4	5	6	7	1	2	3	4	5	6	7

The before-after responses are graphed as a coordinate on a Cartesian graph, with the number at each coordinate indicating the number of respondents with that coordinate combination (Figure 7). Responses on the left side of the graph are interpreted operationally to indicate that the class was ineffective (lower left), or too elementary for the students' level (upper left). Responses on the lower right are interpreted to indicate that the class was effective for individuals professing little prior knowledge, and the upper right effective for students who believe they came in already knowledgeable.

GEOL 200: Evolutionary Systems has the broadest reach, which includes an examination of the role of chaos/complex systems theories in as many complex evolutionary systems across as many disciplines as we can explore. The two classes reported here had a total enrollment of 39 (14 female and 25 male), with 1 freshman, 9 sophomores, 10 juniors, and 19 seniors. About a quarter of the students say they signed up because a friend recommended the course; the remaining usually respond, "It's a gened course and it fit my schedule."

Responses to the assessment questions for GEOL 200 (Figure 7) reflect that overall, the course is effective to very effective. Responses to question 1 indicate that the majority in the class developed a positive commitment and desire to continue to pursue complex systems ideas in the future; the core goal for teaching these ideas. Responses to question 2 indicate that students believe they have learned not only about chaos/complex systems theories, but also know how they fit into larger questions of intellectual history and attempts to find the truth about how the world works.

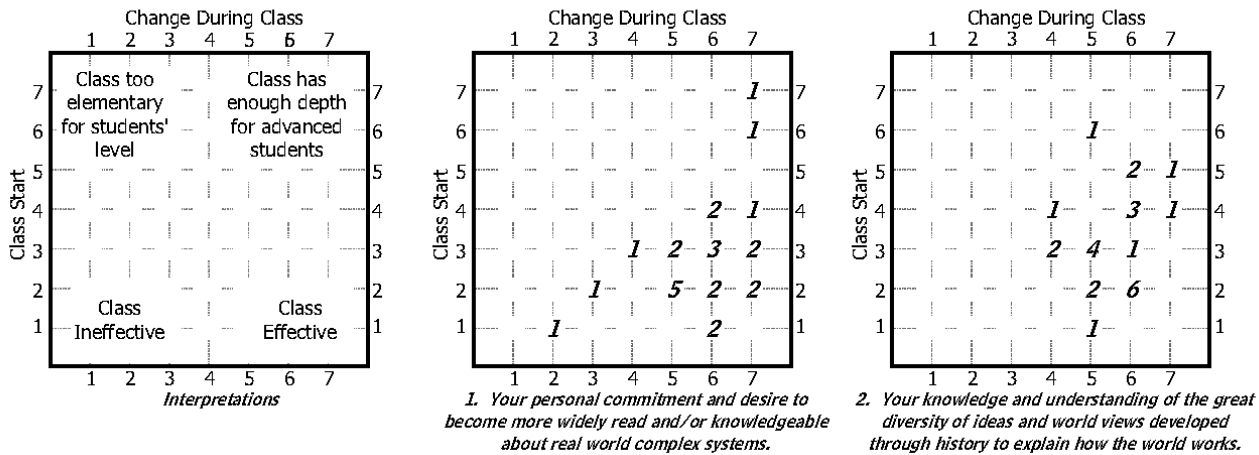
ENVT 200: Environmental Systems Theory is more focussed on the relationship between complex societies and their environments. It was taught for the first time in the Fall, 2009 semester, with only 7 of the 22 being traditional science majors; the remaining majors included accounting, art, management, communication, and public policy, representing a mélange of backgrounds and interests. Class distribution was 9 females and 13 males, with 5 freshmen, 1 sophomore, 8 juniors, and 8 seniors. The first third of the course is dedicated to chaos/complex systems theories following the rubrics in this manuscript, with the addition of exploratory lab experiments for some of the models. The remaining class time examines the structure of ecosystems, ecological successions, extinction causes, Phanerozoic, Pleistocene and Holocene climate/environmental changes, and the rise and fall of complex societies over the past 10,000 years, all as complex systems. As with the GEOL 200 course, the responses to the before/after questions show the course to be effective to highly effective (Figure 7). Responses to question 1 indicate that most students came in with little knowledge of chaos/complex systems and their application to environmental problems but grew substantially. Responses to the other questions show similar improvements. The reliability and validity of these responses are strongly supported by open, written comments offered by students' end-of-course evaluation forms.

Based on these assessment results from the two courses, the major course goals have been met by students. More importantly, however, is the determination of the relative worth of the effort needed to design and deliver the instruction that supports such a unique set of goals. We believe that there is strong evidence to support the assertion that there has been a positive and lasting impact on students' approach to understanding problems in the natural world. In addition, students are capable of comprehending complex systems models in both conceptual and authentic contexts. In doing so, the "value-added" nature of instruction guided by the rubrics is largely established. That said, the range of both the cognitive and affective impacts of the instruction has not yet been fully explored. This remains a focus of continuing work.

CHALLENGES, BENEFITS, AND OPPORTUNITIES FOR TEACHING EVOLUTION AS A COMPLEX SYSTEM

There are challenges, benefits, and opportunities for incorporating, elaborating, self-organizing, and

GEOL 200 - Evolutionary Systems



ENVT 200 - Environmental Systems Theory

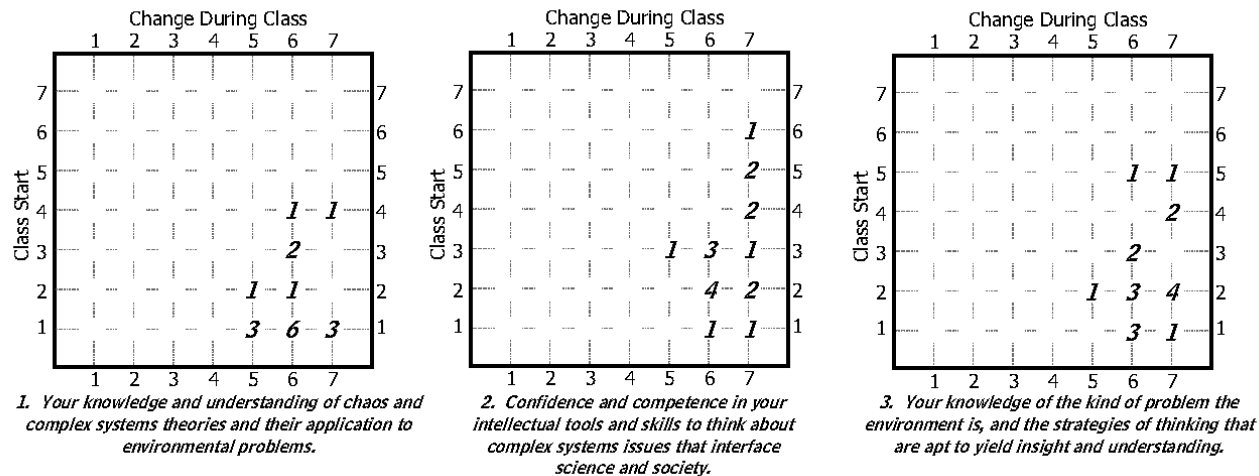


FIGURE 7. Affective assessment results for students' perceptions of their growth in understanding the importance and application of chaos/complex systems theories for two courses. Responses to the assessment questions reflect that overall, the rubrics presented in this paper are effective to very effective at altering students' dispositions. Students report that they are receptive to the ideas, willing to continue to learn about them, apply them, and be influenced by them. GEOL 200: Evolutionary Systems data is combined from two semesters; ENVT 200: Environmental Systems Theory is from one class.

fractionating evolutionary processes into our lesson plans as complex systems. Probably the greatest challenge is that most of us now teaching the Earth sciences have not ourselves been educated in the framework of non-equilibrium concepts (i.e. chaos/complex systems). Because non-equilibrium concepts are unfamiliar or only vaguely familiar, and we do not have a ready framework to put them in, it is hard to know how to start incorporating these ideas in our lesson plans, especially when they challenge our familiar, traditional ways of thinking about and teaching systems.

This leads to the second challenge; the concepts, principles, and mechanisms we propose as ways of thinking about evolutionary processes have been evolving independently, piecemeal, and ever more rapidly in a wide diversity of disciplines: mathematics, physics, chemistry, economics, etc. Historically, the concepts or principles were often developed simultaneously by

individuals in different disciplines, who saw it as a curiosity, without being aware of parallel work in other disciplines, and thereby giving the phenomena a different name. Today concepts in complex systems theory still tend to be diffuse, scattered in many disciplines and a wide diversity of literature sources. The consequence is that although some of these ideas have been around for a while, and some excellent summary books, both popular and technical, are available for particular pieces (e.g. Waldrop, 1992, Johnson, 2002, Sole', 2002, Strogatz, 2004, among others), there still does not exist a coherent or common theoretical framework demonstrating how all these ideas integrate.

On the other hand, one of the benefits of using chaos/complex systems theory principles to explain evolutionary processes is that they cut across all disciplines. What this means is that Earth evolutionary processes can be readily related to processes operating in physical, chemical,

biological, social, and even economic systems (Beinhocker, 2007, Ormerod, 2007). Indeed, what these demonstrate is that there are universal evolutionary processes operating in many realms. We use in our classes examples from many disciplines to illustrate evolution principles, although we use Earth science examples wherever we can.

A second benefit to expanding our teaching of evolutionary mechanisms is more social and political; diffusion of the debate over teaching only Darwinian evolution in the classroom. When evolution is presented as only one theory—Darwinism—taught in a narrow context, disconnected from all the other evolutionary processes that occur all around us all the time, it becomes a lightning rod for controversy. If, instead of beginning with Darwinian theory we begin with a general exploration of the many ways things all around us increase in complexity, diversity, order, and/or interconnectedness (that is, evolve), and develop the three coexisting evolutionary mechanisms to explain them, then Darwinism becomes just one of several ways that things evolve, and Darwinian theory becomes just a special case of elaborating evolution, not *the* one and only theory of evolution. Opponents to the instructional value of evolution must either define very clearly and narrowly what their definition of evolution in fact is, or refute all evolutionary mechanisms as a part of a system.

For the opportunities; we have just begun to develop how complex system mechanisms and their interactions apply to and can be used to explain how Earth systems evolve, individually and conjoined. This is especially true in the realm of fractionating evolution. There are decades of work in every realm of the geosciences to build a theoretical evolutionary foundation based on chaos/complex systems theories to our discipline, and this should please and challenge us as scientists and teachers.

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